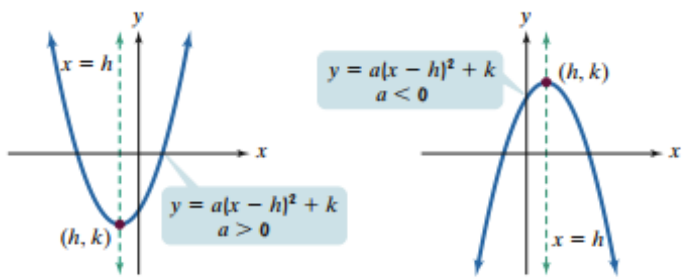
Period 4, April 8, 2025

Graphing $y = a(x - h)^2 + k$ and $y = ax^2 + bx + c$

- If $a > 0$, the graph opens upward. If $a < 0$, the graph opens downward.
- The vertex of $y = a(x - h)^2 + k$ is (h, k).



- The x-coordinate of the vertex of $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$.

For more detail, see Section 2.2, Objective 2.

Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, that is not on the line (see **Figure 9.30**).

Standard Forms of the Equations of a Parabola

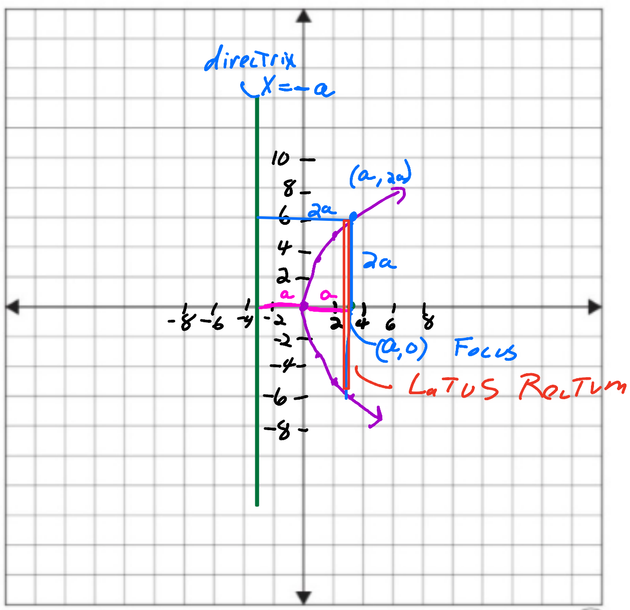
The **standard form of the equation of a parabola** with vertex at the origin is

$$y^2 = 4px \quad \text{or} \quad x^2 = 4py.$$

Figure 9.32(a) at the top of the next page illustrates that for the equation on the left, the focus is on the x-axis, which is the axis of symmetry. **Figure 9.32(b)** illustrates that for the equation on the right, the focus is on the y-axis, which is the axis of symmetry.

$y^2 = 4px$ $p = 3$ Focal Length

Find the focus and directrix of the parabola given by $y^2 = 12x$. Then graph the parabola.



X	Y
0	0
1	$\pm\sqrt{12} = \pm 2\sqrt{3} = \pm 3.46$
2	$\pm\sqrt{24} = \pm 4.9$
3	$\pm\sqrt{36} = \pm 6$

vertex is halfway between Focus and directrix.

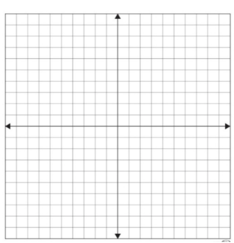
$y^2 = 12x$

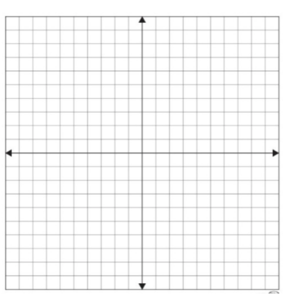
$(2a)^2 = 12a$

$\frac{4a^2}{4a} = \frac{12a}{4a}$

$a = 3$

Focal Length $= 3$





The Latus Rectum and Graphing Parabolas

The **latus rectum** of a parabola is a line segment that passes through its focus, is parallel to its directrix, and has its endpoints on the parabola. **Figure 9.34** shows that the length of the latus rectum for the graphs of $y^2 = 4px$ and $x^2 = 4py$ is $|4p|$.

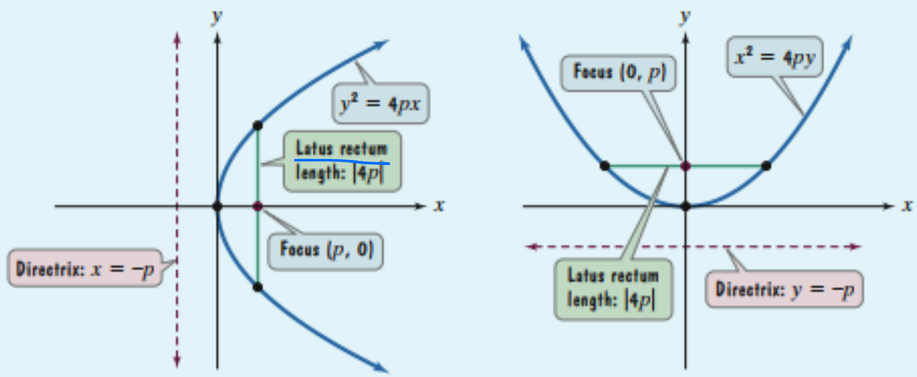
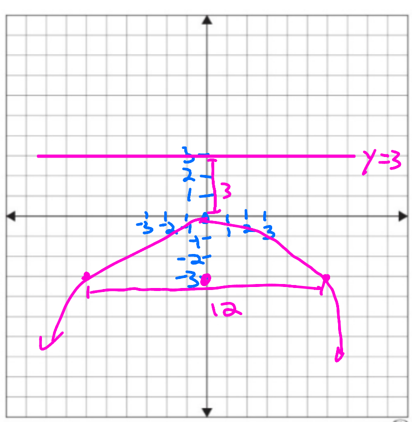


Figure 9.34 Endpoints of the latus rectum are helpful in determining a parabola's "width," or how it opens.

CHECK POINT 2 Find the focus and directrix of the parabola given by $x^2 = -12y$. Then graph the parabola.



opens Down

y must be negative

Vertex (0,0)

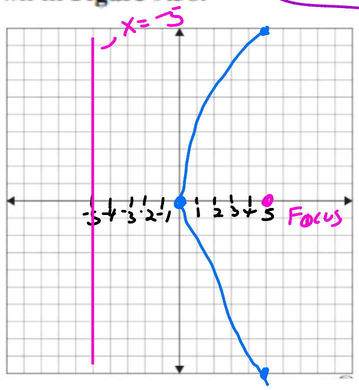
Focal Length $\frac{12}{4} = 3$

Focus is in The Parabola

directrix is Line outside Parabola

Focus (0,-3)

Find the standard form of the equation of a parabola with focus (5, 0) and directrix $x = -5$, shown in **Figure 9.36**.



Focal LengTh = 5 = P

Vertex (0,0)

LaTus = 20 = 4·Focal LengTh = 4·5

$Y^2 = +4P \cdot X = 20X$

Equation	Vertex	Axis of Symmetry	Focus	Directrix	Description
$(y - k)^2 = 4p(x - h)$	(h, k)	Horizontal	$(h + p, k)$	$x = h - p$	If $p > 0$, opens to the right. If $p < 0$, opens to the left.
$(x - h)^2 = 4p(y - k)$	(h, k)	Vertical	$(h, k + p)$	$y = k - p$	If $p > 0$, opens upward. If $p < 0$, opens downward.

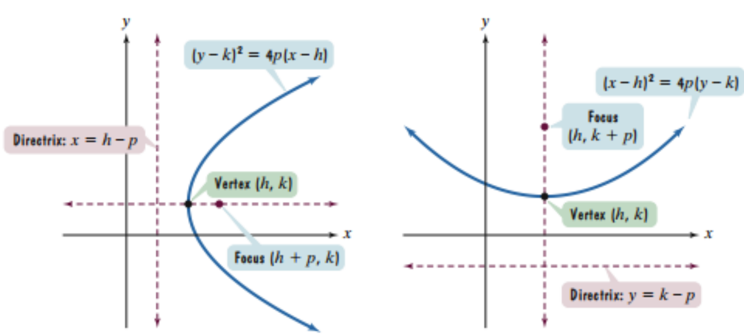


Figure 9.37 Graphs of parabolas with vertex at (h, k) and $p > 0$

Find the vertex, focus, and directrix of the parabola given by

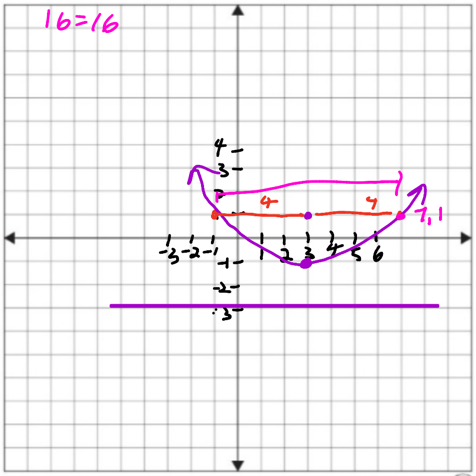
$(7-3)^2=8(1+1)$

$$(x-3)^2 = 8(y+1).$$ $\Rightarrow$ opens up

$4^2=8\cdot 2$

$16=16$

Vertex $(3,-1)$



Focal Length $=\frac{8}{4}=2$

Focus up 2 from vertex $(3,1)$

directrix down 2 from vertex

$y=-3$

Latus $=8$

Find the vertex, focus, and directrix of the parabola given by

$$y^2 + 2y + 12x - 23 = 0.$$

$-12x + 23 \qquad -12x + 23$

Then graph the parabola.

$y^2 + 2y + 1 = -12x + 23 + 1$

$a = 1$

$b = 2$

$\frac{b}{2} = \frac{2}{2} = 1$

$\left(\frac{b}{2}\right)^2 = (1)^2 = 1$

$(y+1)^2 = -12x + 24$

$(y+1)^2 = -12(x-2)$ opens LeFT

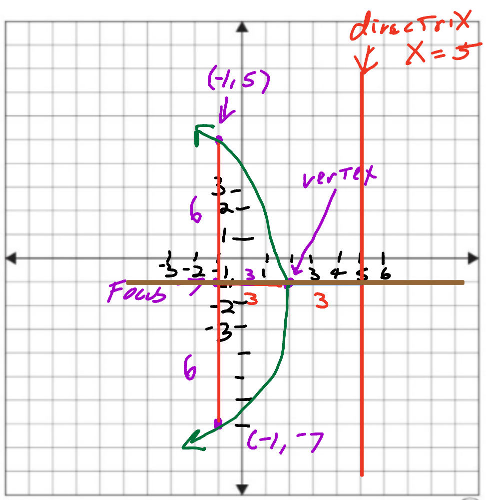
VerTex $= (2, -1)$

Focal LengTh $\frac{12}{4} = 3$

LaTus $= 12$

axis Symm $y = -1$ (Through Focus and VerTex)

directrix



Identifying a Conic Section without Completing the Square

A nondegenerate conic section of the form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0,$$

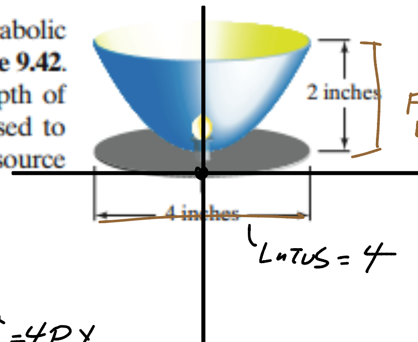
in which A and C are not both zero, is

- a circle if $A = C$,
- a parabola if $AC = 0$,
- an ellipse if $A \neq C$ and $AC > 0$, and
- a hyperbola if $AC < 0$.

Identify the graph of each of the following nondegenerate conic sections:

a. $4x^2 - 25y^2 - 24x + 250y - 489 = 0$
b. $x^2 + y^2 + 6x - 2y + 6 = 0$
c. $y^2 + 12x + 2y - 23 = 0$
d. $9x^2 + 25y^2 - 54x + 50y - 119 = 0.$

An engineer is designing a flashlight using a parabolic reflecting mirror and a light source, shown in **Figure 9.42**. The casting has a diameter of 4 inches and a depth of 2 inches. What is the equation of the parabola used to shape the mirror? At what point should the light source be placed relative to the mirror's vertex?



Focal Length = 1

Latus = 4

$x^2 = 4py$

$x^2 = 4(1) \cdot y$

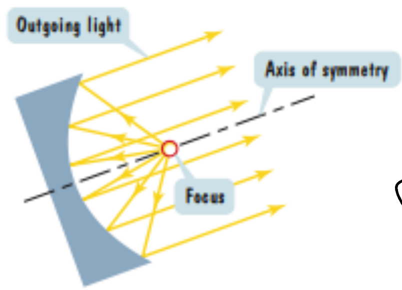


Figure 9.40(a) Parabolic surface reflecting light

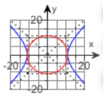
Find the solution set for the system below by graphing both of the system's equations in the same rectangular coordinate system and finding points of intersection. Check all solutions in both equations.

$x^2 - y^2 = 121$ → Hyperbola (opens LEFT/Right

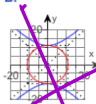
$x^2 + y^2 = 121$ — Circle

Graph both of the system's equations in the same rectangular coordinate system. Choose the correct graph below.

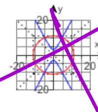
A.



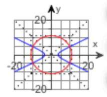
B.

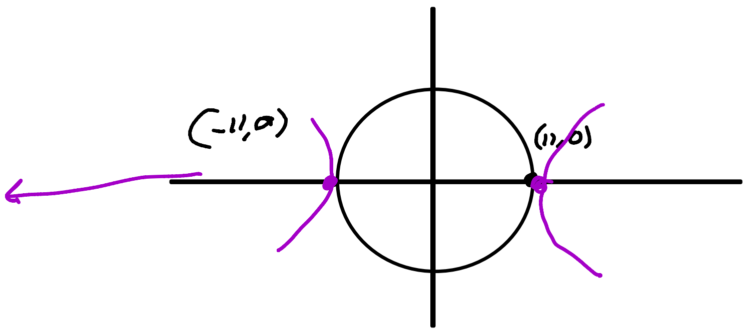


C.



D.





$\frac{x^2 - y^2}{121} = \frac{121}{121}$

$\frac{x^2}{121} - \frac{y^2}{121} = 1$

x	y
11	0 ← $\frac{121}{121} - \frac{y^2}{121} = 1$
−11	0

Use the exponential decay model, $A = A_0 e^{kt}$, to solve the following.

The half-life of a certain substance is 16 years. How long will it take for a sample of this substance to decay to 88% of its original amount?

It will take approximately ☐ [▼] for the sample of the substance to decay to 88% of its original amount.

(Round the final answer to one decimal place as needed. Round all intermediate values to four decimal places as needed.)

100 when $T=0$ → $100 = A_0 e^{k\cdot 0}$ ⇒ $100 = A_0 e^0$ ⇒ $100 = A_0 \cdot 1$

$A_0 = 100$

50 when $T=16$

$A = 100e^{kT}$

$\frac{50}{100} = \frac{\cancel{100}e^{k\cdot 16}}{\cancel{100}}$

$\ln\frac{1}{2} = \ln e^{16k}$

$\frac{\ln\frac{1}{2}}{16} = \frac{\cancel{16}k\cdot\cancel{\ln e}}{\cancel{16}}$

$\frac{\ln\frac{1}{2}}{16} = -0.043321699$

$A = 100e^{-0.0433217T}$

$\frac{88}{100} = \frac{\cancel{100}e^{-0.0433217T}}{\cancel{100}}$

$\ln .88 = \ln e^{-0.0433217T}$

$\frac{-0.1278334}{-0.0433217} = \frac{-0.0433217T\cancel{\ln e}}{-0.0433217}$

Time = 2.95 years

Find the standard form of the equation of the hyperbola satisfying the given conditions.

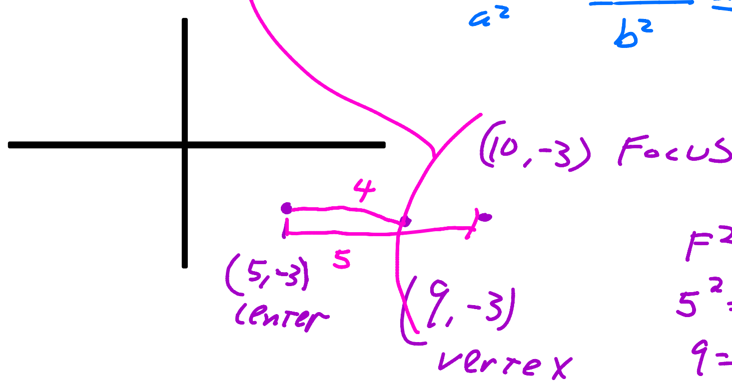
Center: (5, − 3); focus: (10, − 3); vertex: (9, − 3)

The equation is $\square$.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$ Open Left/Right

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$ opens up/Down

$$\frac{(x-5)^2}{4^2} - \frac{(y+3)^2}{3^2} = 1$$



$F^2 = a^2 + b^2$

$5^2 = 4^2 + b^2$

$9 = b^2 \Rightarrow b = 3$

Convert the equation to standard form by completing the square on x and y. Then graph the hyperbola. Locate the foci and find the equations of the asymptotes.

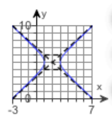
$x^2 - y^2 - 4x - 10y - 22 = 0$

The standard form of the equation is $\frac{(x-2)^2}{1} - \frac{(y+5)^2}{1} = 1$.

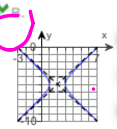
(Type an equation.)

Graph the hyperbola. Choose the correct graph below.

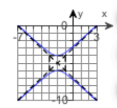
A.



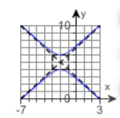
B.



C.



D.



The foci is/are $(\sqrt{2}+2,-5),(-\sqrt{2}+2,-5)$.

(Type an ordered pair. Type an exact answer, using radicals as needed. Use a comma to separate answers as needed.)

The equation of the asymptote with the positive slope is $y = x - 7$.

The equation of the asymptote with the negative slope is $y = -x - 3$.

(Simplify your answers. Use integers or fractions for any numbers in the equation.)

$x^2 - y^2 - 4x - 10y - \cancel{22} = 0$

$+22 \qquad +22$

opens Left Right

$x^2 - 4x + 4 \quad -y^2 - 10y \quad = 22 + 4 - 25 = 1$

$a = 1$

$b = -4$

$\frac{b}{2} = -\frac{4}{2} = -2$

$\left(\frac{b}{2}\right)^2 = (-2)^2 = 4$

$-1(y^2 + 10y + 25)$

$a = 1$

$b = 10$

$\frac{b}{2} = \frac{10}{2} = 5$

$\left(\frac{b}{2}\right)^2 = (5)^2 = 25$

Foci$^2 = 1^2 + 1^2$

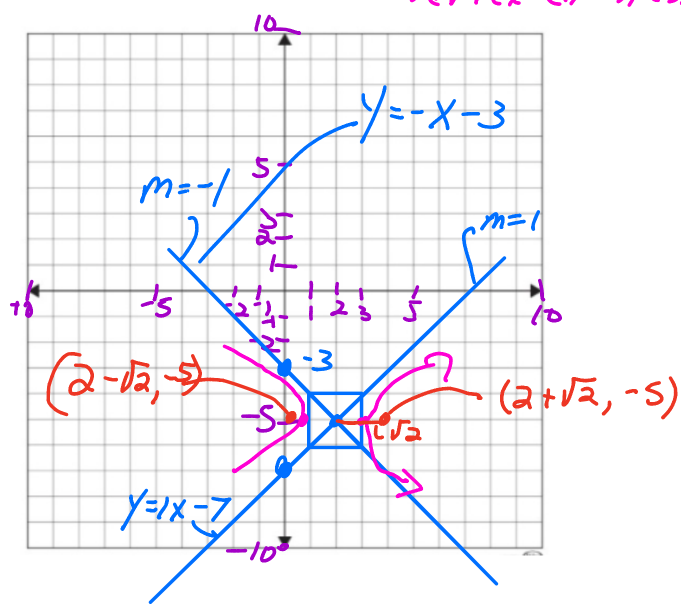
$F = \sqrt{2}$

vertex (1,-5), (3,-5)

$(x-2)^2 - (y+5)^2 = 1$

Center (2,-5)

$\frac{(x-2)^2}{1} - \frac{(y+5)^2}{1} = 1$

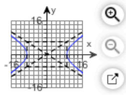


Use vertices and asymptotes to graph the hyperbola. Locate the foci and find the equations of the asymptotes.

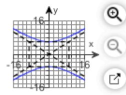
$81y^2 - 36x^2 = 2916$

Graph the hyperbola. Choose the correct graph below.

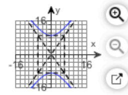
A.



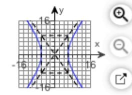
B.



C.



D.



The foci is/are at the point(s) $(0, -3\sqrt{13}), (0, 3\sqrt{13})$.
(Type an ordered pair. Type an exact answer, using radicals as needed. Use a comma to separate answers as needed.)

The equation of the asymptote with the positive slope is $y = \frac{2}{3}x$. The equation of the asymptote with the negative slope is $y = -\frac{2}{3}x$.
(Simplify your answers. Use integers or fractions for any numbers in the equation.)

$m = \frac{6}{9} = \frac{2}{3}$

$b = 0$

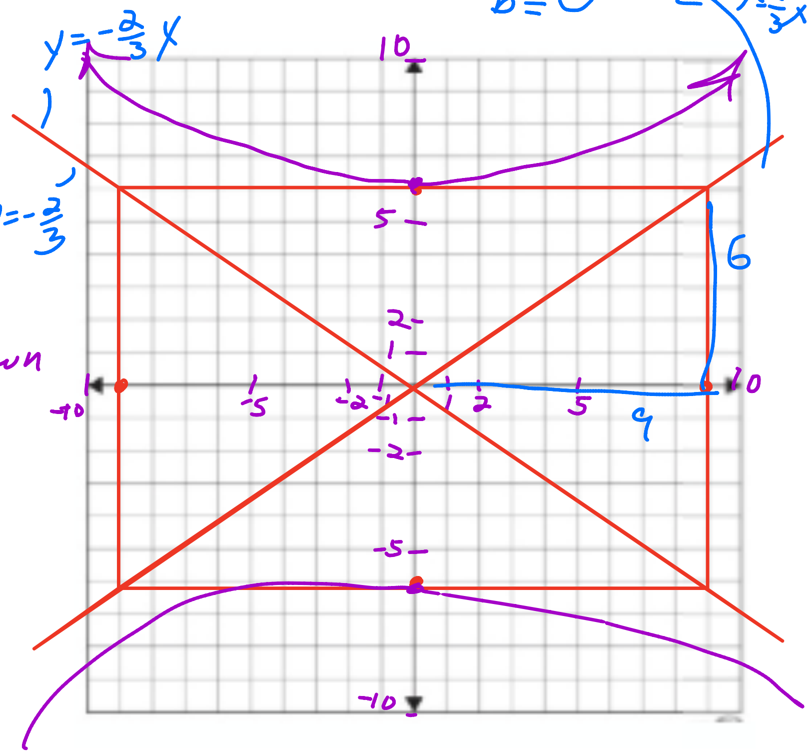
$$\frac{81y^2 - 36x^2}{2916} = \frac{2916}{2916}$$

$m = -\frac{2}{3}$

$$\frac{81y^2}{2916} - \frac{36x^2}{2916} = 1$$

$$\frac{y^2}{36} - \frac{x^2}{81} = 1$$ opens up/Down

Center(0, 0)



$\sqrt{36} = 6$

$\sqrt{81} = 9$

$F^2 = 9^2 + 6^2 = 81 + 36 = 117$

$F = \sqrt{117}$

$(0, \sqrt{117})$ and $(0, -\sqrt{117})$

An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 receives the sound 1 second before microphone M_2. Assuming sound travels at 1100 feet per second, determine the possible locations of the explosion relative to the locations of the microphones.

Which equation below describes the possible locations of the explosion relative to the locations of the microphones?

1 mile = 5280 Feet

○ A. $\frac{x^2}{5280} - \frac{y^2}{550} = 1$

○ B. $\frac{x^2}{27,575,900} - \frac{y^2}{302,500} = 1$

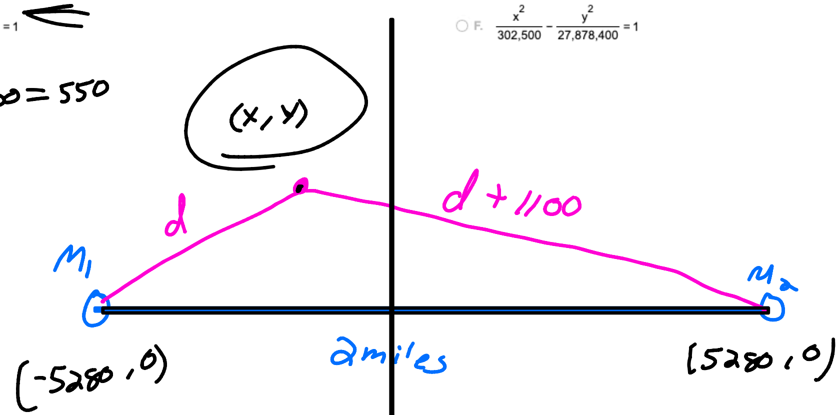
○ C. $\frac{x^2}{27,878,400} - \frac{y^2}{302,500} = 1$

○ D. $\frac{x^2}{550} - \frac{y^2}{5280} = 1$

✔ E. $\frac{x^2}{302,500} - \frac{y^2}{27,575,900} = 1$ ⟵

○ F. $\frac{x^2}{302,500} - \frac{y^2}{27,878,400} = 1$

$\sqrt{302,500} = 550$



$$d = \sqrt{(x-(-5280))^2 + (y-0)^2}$$

$$d + 1100 = \sqrt{(x-5280)^2 + (y-0)^2}$$

$$\left(\sqrt{(x+5280)^2 + y^2} + 1100\right)^2 = \left(\sqrt{(x-5280)^2 + y^2}\right)^2$$

$$\left(\sqrt{(x+5280)^2 + y^2} + 1100\right)\left(\sqrt{(x+5280)^2 + y^2} + 1100\right)$$

$$(x+5280)^2 + \cancel{y^2} + 2200\sqrt{(x+5280)^2 + y^2} + 1210000 = (x-5280)^2 + \cancel{y^2}$$

$$\cancel{x^2} + 10560x + \cancel{27878400} + 2200\sqrt{(x+5280)^2 + y^2} + 1210000 = \cancel{x^2} - 10560x + \cancel{27878400}$$

$$\frac{2200\sqrt{(x+5280)^2 + y^2}}{2200} = \frac{-21120x - 1210000}{2200}$$

$$\left(\sqrt{(x+5280)^2 + y^2}\right)^2 = (-9.6x - 550)^2$$

$$(x+5280)^2 + y^2 = 92.16x^2 + 10560x + 302500$$

$$x^2 + \cancel{10560x} + 27875400 + y^2 = 91.16x^2 + \cancel{10560x} + 302500$$

$$\frac{27575900}{27575900} = \frac{91.16x^2 - y^2}{27575900}$$

$$1 = \frac{91.16x^2}{27575900} - \frac{y^2}{27575900}$$

$$1 = \frac{x^2}{302500} - \frac{y^2}{27575900}$$

$$\frac{9x^2 + y^2}{36} = \frac{36}{36}$$

$$\frac{9x^2}{36} + \frac{y^2}{36} = 1$$

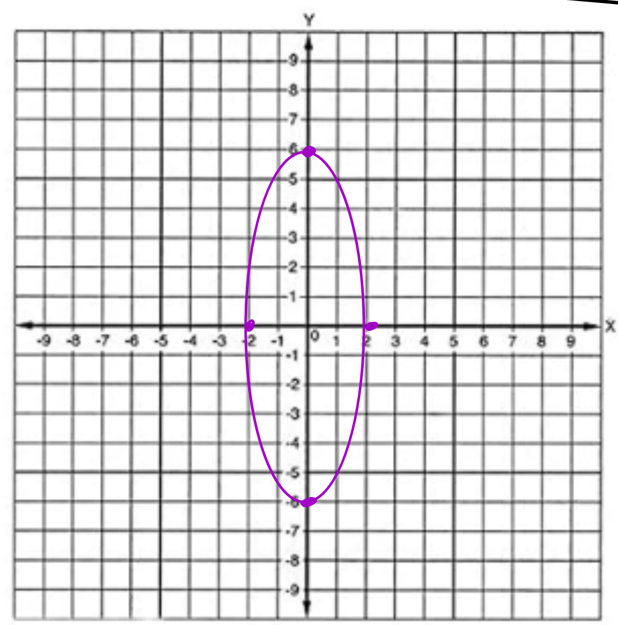
$$\frac{x^2}{4} + \frac{y^2}{36} = 1$$

Right/Left 2

Up/Down 6

$\sqrt{4} = 2$

$\sqrt{36} = 6$



(0,6)
(0,-6)

$$\frac{4x^2 + y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{4} + \frac{y^2}{6} = 1$$

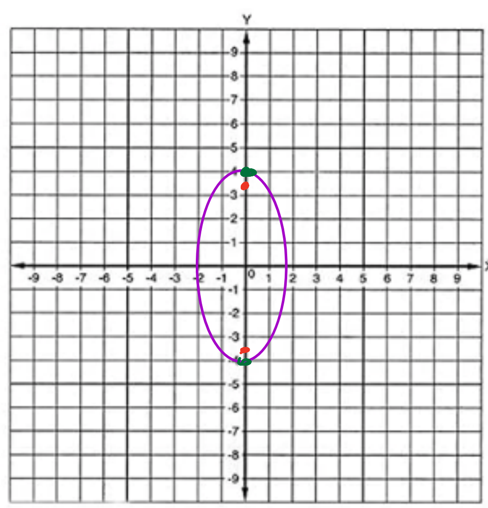
$16 = F^2 + 4$

$12 = F^2$

$\pm\sqrt{12} = F$

Verticies
(0,4)
and
(0,-4)

Foci (0, $\sqrt{12}$)
and
(0, $-\sqrt{12}$)



$4x^2 - 24x + 9y^2 + 90y + 225 = 0$

$4(x^2 - 6x + 9\quad) + 9(y^2 + 10y + 25) = -225 + 9 \cdot 4 + 25 \cdot 9$

$b = -6$ $\quad a = 1$

$\quad b = 10$

$\frac{b}{2} = -3$ $\quad \frac{b}{2} = 5$

$\left(\frac{b}{2}\right)^2 = 9$ $\quad \left(\frac{b}{2}\right)^2 = 25$

$4(x-3)^2 + 9(y+5)^2 = \cancel{-225} + 36 + \cancel{225}$

$\frac{4(x-3)^2}{36} + \frac{9(y+5)^2}{36} = 1$

$\frac{(x-3)}{9} + \frac{(y+5)}{4} = 1$